Time series econometrics on crypto currency data

Load data

I'll use Ethereum AUD as an example for now. See https://www.ethereum.org/ for more info.

"Unlike the Bitcoin blockchain which is essentially a list of the transactions on the network, the Ethereum blockchain contains both the list of transactions and the state of the blockchain (in the simplest case, the balances of all the accounts). The Ethereum whitepaper provides a list of potential use cases for the platform, but the primary one is the ability to build 'smart contracts' that will execute themselves, eliminating the need for counterparties to rely on each others trustworthiness. ... There are two major differences between Ether and Bitcoin. First, unlike Bitcoin who's total supply is capped at 21 million units, there is no fixed supply of Ether. Secondly, blocks are added to the Ether blockchain every 15 seconds (on average), unlike the approximately 10 minutes for Bitcoin." – See this dude's thesis.

In [123…

```
#!pip install yfinance
import yfinance as yf
import datetime as dt
import pandas as pd

start = dt.datetime(2022,12,1)
end = dt.datetime(2023,12,2)

df = yf.download('ETH-AUD', start, end)
```

```
[*********************100%%**********************]  1 of 1 completed
```

In [124…

```
df.head()
```

Out[124…

	Open	High	Low	Close	Adj Close	Volume
Date						
2022-12-01	1905.750366	1902.465576	1864.194458	1873.937744	1873.937744	10069424071
2022-12-02	1874.018066	1904.564819	1863.372070	1904.564819	1904.564819	9143367234
2022-12-03	1904.790161	1912.712524	1826.441895	1829.241577	1829.241577	8345758516
2022-12-04	1829.071289	1884.089722	1828.619019	1879.173462	1879.173462	7594637486
2022-12-05	1878.795044	1915.492432	1869.946289	1878.171631	1878.171631	9125425447

In [125…

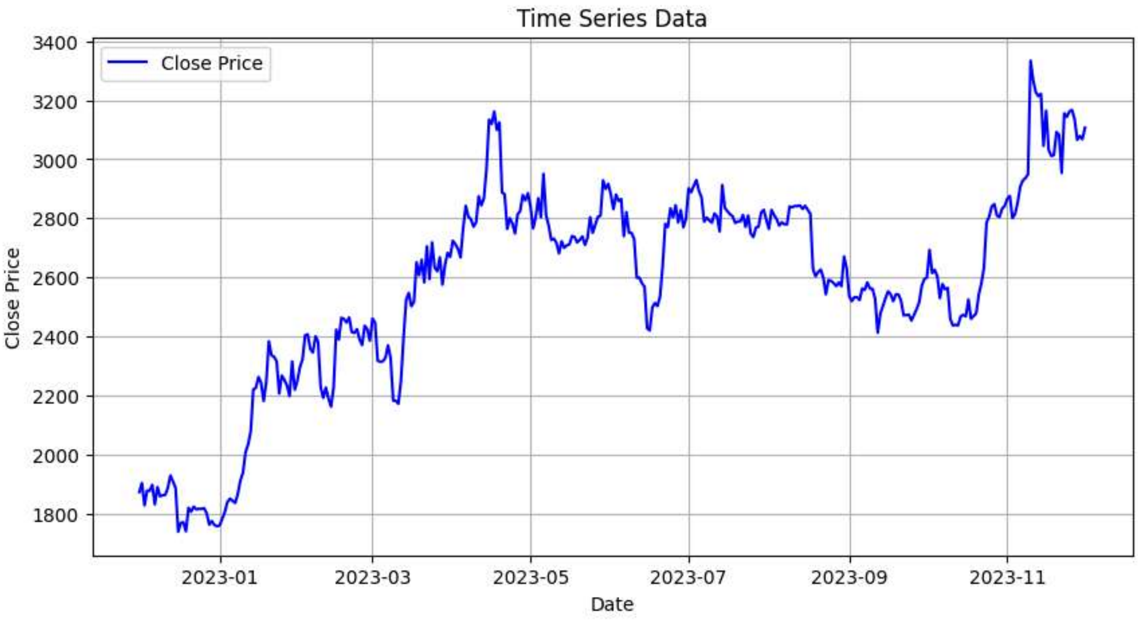
```
df.tail()
```

Out[125…

	Open	High	Low	Close	Adj Close	Volume
Date						
2023-11-27	3166.480225	3181.549561	3097.235596	3135.169678	3135.169678	12239314271
2023-11-28	3133.809082	3147.625732	3009.053955	3065.574463	3065.574463	15989736738
2023-11-29	3065.725586	3118.075195	3017.007324	3079.087402	3079.087402	14890518016
2023-11-30	3078.860107	3125.369385	3057.944336	3067.608643	3067.608643	13517824040
2023-12-01	3066.746094	3109.025391	3053.591797	3106.890625	3106.890625	12272509114

In [126…

```
import matplotlib.pyplot as plt

plt.figure(figsize=(10, 5))
plt.plot(df.index, df['Close'], linestyle='-', color='b', label='Close Price'
plt.title('Time Series Data')
plt.xlabel('Date')
plt.ylabel('Close Price')
plt.legend()
plt.grid(True)
plt.show()
```



Stationarity

We see ETH-AUD has a general upward trend over the year.

To calculate reliable test statastics, we need stationarity in our data. Stationarity is a

To calculate reliable test statistics, we need stationarity in our data. Stationarity is a fundamental assumption in many time series models as it simplifies the modeling process and allows for the use of standard statistical techniques. When your data is stationary, you can make meaningful inferences about the relationships between variables and make reliable forecasts. Non-stationary data can make it challenging to estimate model parameters and can lead to unstable and unreliable results.

Stationarity is where a time series has a constant mean and constant variance over time. From a purely visual assessment, time plots that do not show trends or seasonality can be considered stationary.

Transformations

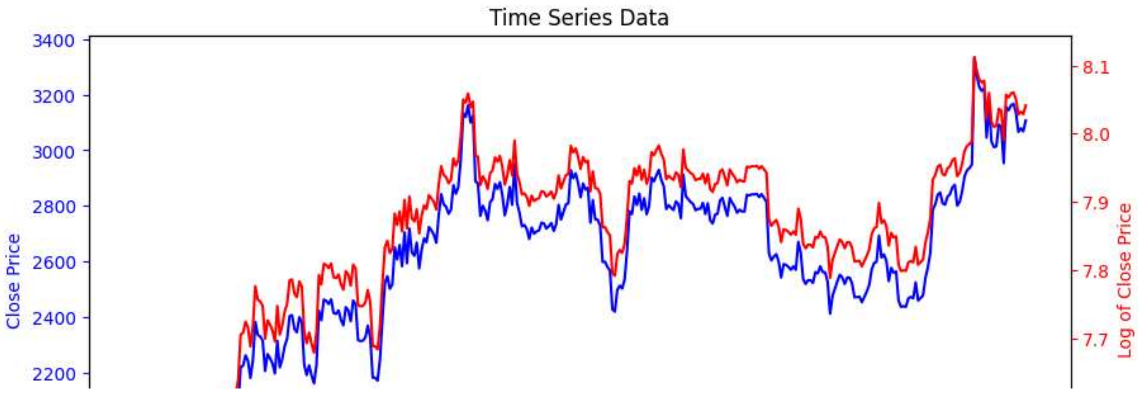
We can transform the time series to be stationary by:

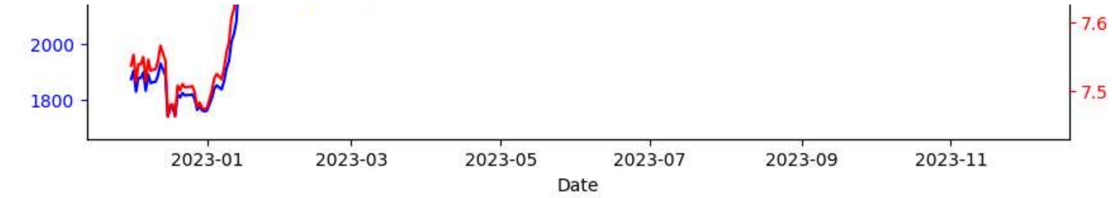
1. Taking the log;
2. Using the growth rate; or
3. Differencing over time (taking the log of today and minus it by the log of yesterday).

I'll start by taking the log:

In [127…

```
# Taking the Log
import numpy as np

df['Log_Close'] = np.log(df['Close'])

# Plotting
fig, ax1 = plt.subplots(figsize=(10, 5))

# Plot Close Price on the primary y-axis
ax1.plot(df.index, df['Close'], linestyle='-', color='b', label='Close Price'
ax1.set_xlabel('Date')
ax1.set_ylabel('Close Price', color='b')
ax1.tick_params('y', colors='b')

# Create a secondary y-axis for Log of Close Price
ax2 = ax1.twinx()
ax2.plot(df.index, df['Log_Close'], linestyle='-', color='r', label='Log of C
ax2.set_ylabel('Log of Close Price', color='r')
ax2.tick_params('y', colors='r')

plt.title('Time Series Data')
plt.show()
```





Taking the log makes no difference (no pun intended).

Now I'll try the logged growth rate:

In [128…

```
# Calculate the daily percentage change (growth rate)
df['Close_Log_Growth_Rate'] = df['Log_Close'].pct_change() * 100

# Plotting growth rate on its own
plt.figure(figsize=(10, 5))
plt.plot(df.index, df['Close_Log_Growth_Rate'], linestyle='-', color='g', lab
plt.title('Close Price Growth Rate Over Time')
plt.xlabel('Date')
plt.ylabel('Close Growth Rate (%)')
plt.legend()
plt.grid(True)
plt.show()
```

That looks like a legit stationary process.

Testing for stationarity

We can test for stationarity by:

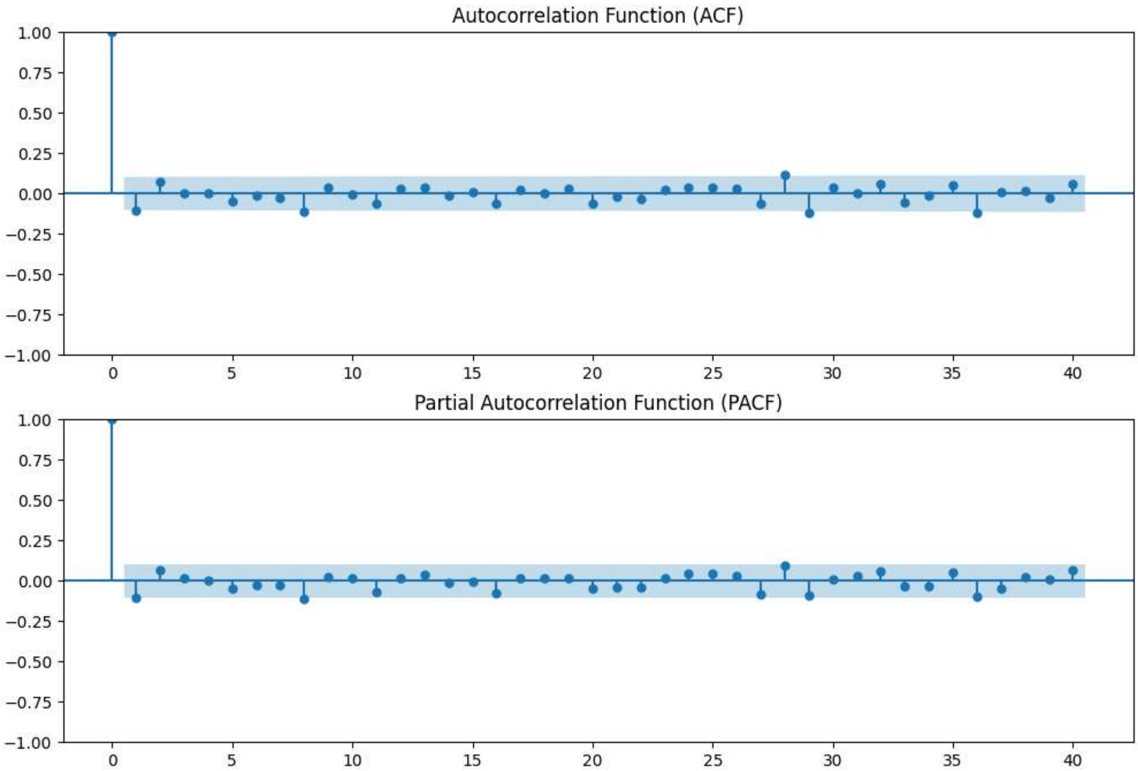
1. plotting the Auto-Correlation Function, and Partial Auto-Correlation Function (ACF and PACF respectively); or
2. using the Augmented Dickey-Fuller (ADF) test.

We first plot the ACF and PACF:

In [129…

```
import statsmodels.api as sm
from statsmodels.graphics.tsaplots import plot_acf, plot_pacf
```

```python
from statsmodels.graphics.tsaplots import plot_acf, plot_pacf
import matplotlib.pyplot as plt

df = df.dropna()  # Drop the first row with NaN value

# Plot ACF and PACF
fig, (ax1, ax2) = plt.subplots(2, 1, figsize=(12, 8))

# ACF plot starting at lag 1
plot_acf(df['Close_Log_Growth_Rate'], lags=40, ax=ax1)
ax1.set_title('Autocorrelation Function (ACF)')

# PACF plot starting at lag 1
plot_pacf(df['Close_Log_Growth_Rate'], lags=40, ax=ax2)
ax2.set_title('Partial Autocorrelation Function (PACF)')

plt.show()
```



We find for ETH that there is significant autocorrelation and partial autocorrelation at the first lag. In other words, the value of the time series at a given time is highly correlated with its value one time step prior.

The presence of significant autocorrelation in the first lag could imply that the time series data is not a white noise process, and there may be some serial dependence or memory effect in the data.

We now test stationarity using the ADF test. If the p-value from the ADF test is below a certain significance level (commonly 0.05), you may reject the null hypothesis and conclude that the series is stationary.

In [130...

```python
import statsmodels.api as sm

# Perform ADF test
result = sm.tsa.adfuller(df['Close_Log_Growth_Rate'])

# Extract and print the results
```

```
# Extract and print the results
adf_statistic = result[0]
p_value = result[1]

print(f'ADF Statistic: {adf_statistic}')
print(f'p-value: {p_value}')

# Interpret the results
if p_value <= 0.05:
    print('Reject the null hypothesis. The series is likely stationary.')
else:
    print('Fail to reject the null hypothesis. The series may not be stationa
```

```
ADF Statistic: -21.18197594732444
p-value: 0.0
Reject the null hypothesis. The series is likely stationary.
```

Looks like its all stationary.

Autoregressive Moving Average Modelling

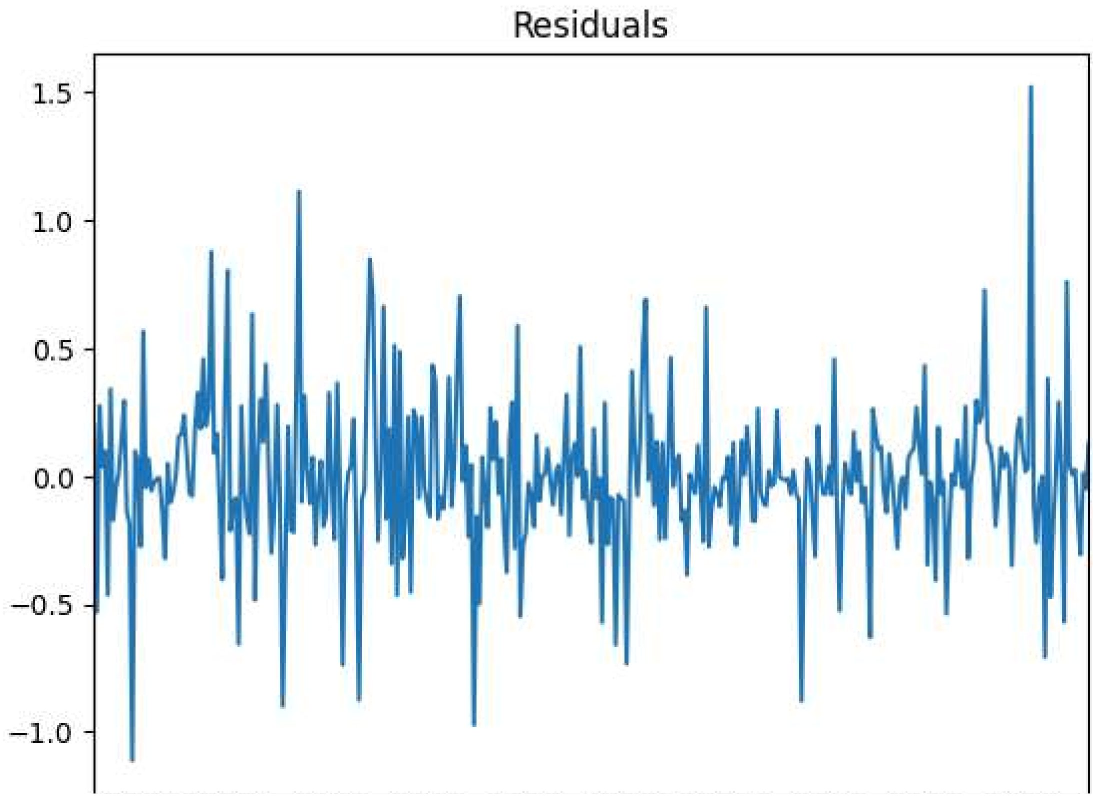
Given the ACF and PACF of the first-order differenced ETH have significant first lags for both the ACF and PACF, we can use an ARMA(1,1) to model ETH.

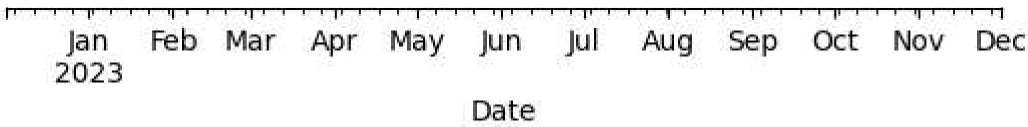
In [131…

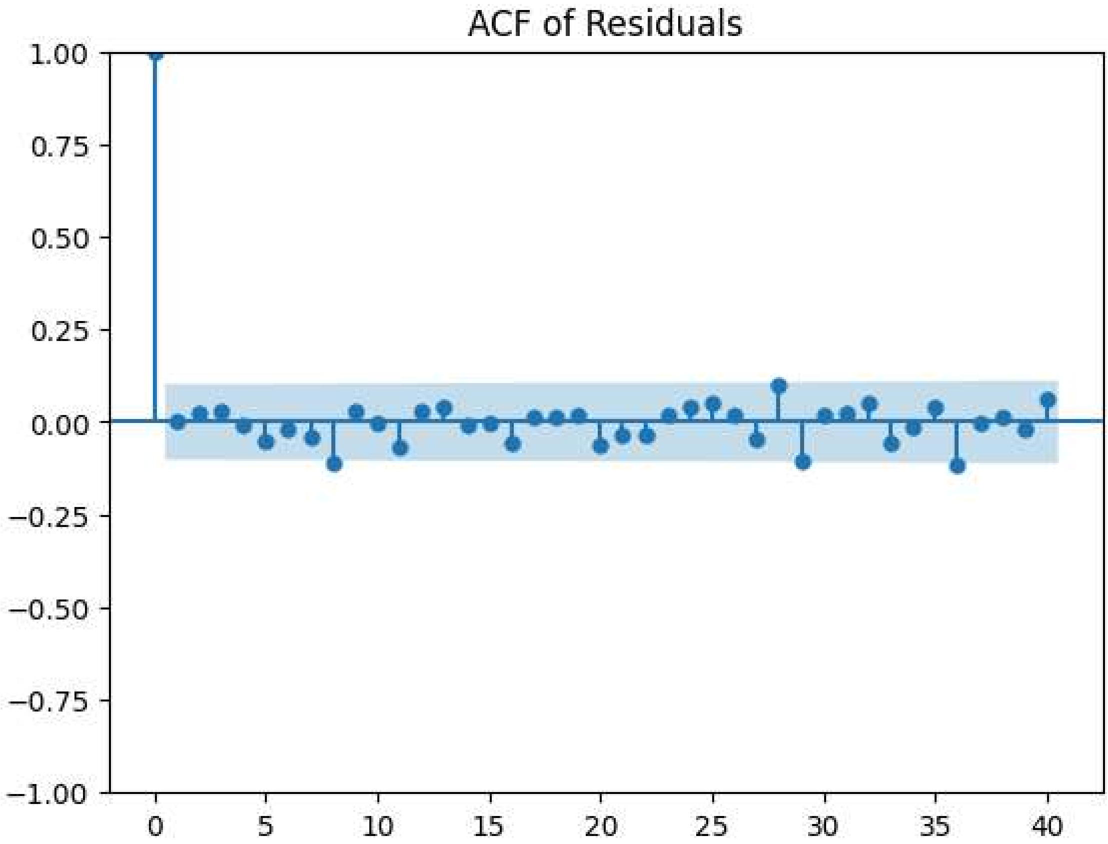
```
import statsmodels.api as sm

# Assuming df['Close_Log_Growth_Rate'] is your time series data

# Create and fit the ARMA(1,1) model
model = sm.tsa.ARIMA(df['Close_Log_Growth_Rate'], order=(1, 0, 1))
results = model.fit()

# Print the model summary
print(results.summary())

# Plot the residuals
residuals = results.resid
residuals.plot(title="Residuals")
plt.show()

# Plot the autocorrelation of residuals
sm.graphics.tsa.plot_acf(residuals, lags=40)
plt.title("ACF of Residuals")
plt.show()

# Plot the partial autocorrelation of residuals
sm.graphics.tsa.plot_pacf(residuals, lags=40)
plt.title("PACF of Residuals")
plt.show()
```

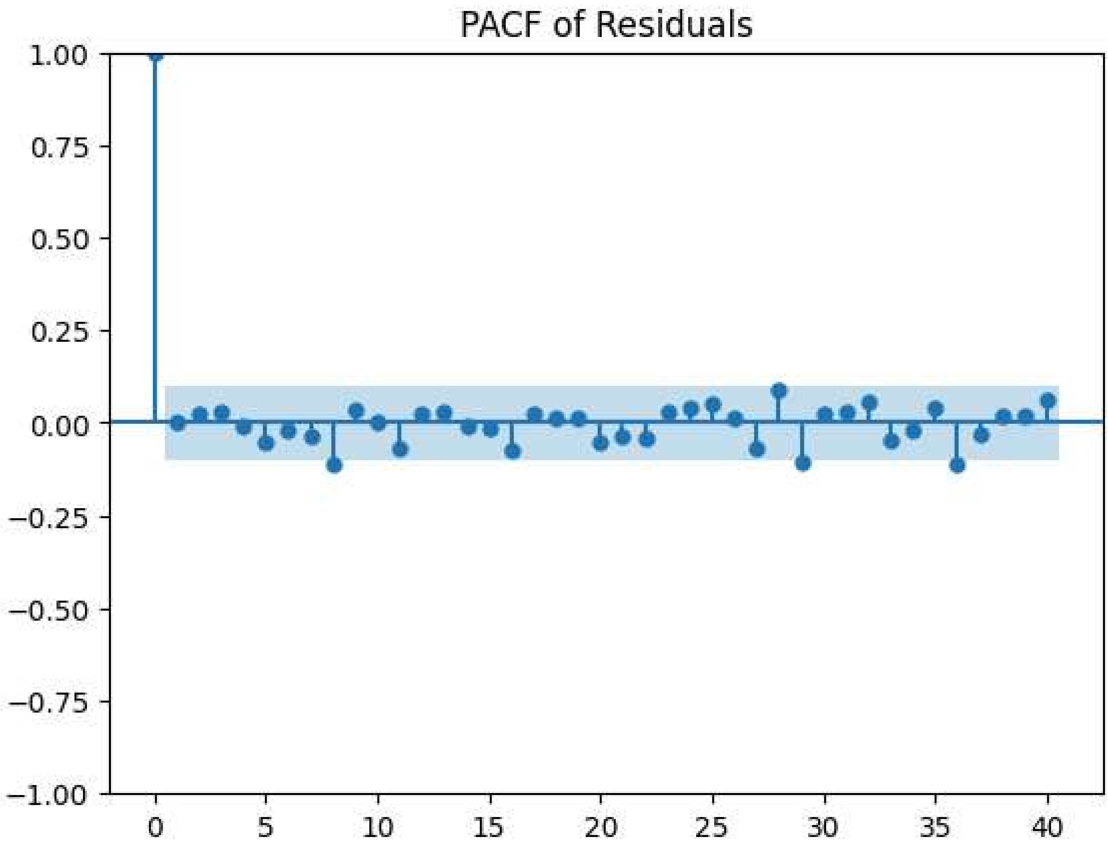
```
/Library/Frameworks/Python.framework/Versions/3.11/lib/python3.11/site-package
s/statsmodels/tsa/base/tsa_model.py:473: ValueWarning: No frequency information
was provided, so inferred frequency D will be used.
  self._init_dates(dates, freq)
/Library/Frameworks/Python.framework/Versions/3.11/lib/python3.11/site-package
s/statsmodels/tsa/base/tsa_model.py:473: ValueWarning: No frequency information
was provided, so inferred frequency D will be used.
  self._init_dates(dates, freq)
/Library/Frameworks/Python.framework/Versions/3.11/lib/python3.11/site-package
s/statsmodels/tsa/base/tsa_model.py:473: ValueWarning: No frequency information
```

```
was provided, so inferred frequency D will be used.
  self._init_dates(dates, freq)
                               SARIMAX Results
================================================================================
==
Dep. Variable:     Close_Log_Growth_Rate   No. Observations:                  3
65
Model:                      ARIMA(1, 0, 1)   Log Likelihood                -75.4
88
Date:                     Mon, 25 Dec 2023   AIC                           158.9
77
Time:                             16:18:31   BIC                           174.5
76
Sample:                         12-02-2022   HQIC                          165.1
76
                              - 12-01-2023
Covariance Type:                       opg
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.0181      0.015      1.232      0.218      -0.011      0.047
ar.L1         -0.5006      0.389     -1.289      0.198      -1.262      0.261
ma.L1          0.3962      0.411      0.964      0.335      -0.410      1.202
sigma2         0.0885      0.004     21.106      0.000       0.080      0.097
================================================================================
====
Ljung-Box (L1) (Q):                   0.01   Jarque-Bera (JB):                17
7.43
Prob(Q):                              0.93   Prob(JB):
0.00
Heteroskedasticity (H):               0.62   Skew:
0.31
Prob(H) (two-sided):                  0.01   Kurtosis:
6.36
================================================================================
====

Warnings:
[1] Covariance matrix calculated using the outer product of gradients (complex-
step).
```









Looks like the first lag of AR and MA are not significant (they fall within the confidence intervals and do not have p values less than 0.05).

Regression model with time series errors

We know that smaller coins often follow the fluctuations of Bitcoin. In other words, the price of ETH hevily depends on BTC. Given this information, we can model a regression and then add AR or MA errors to properly model the fluctuations.

Lets confirm this:

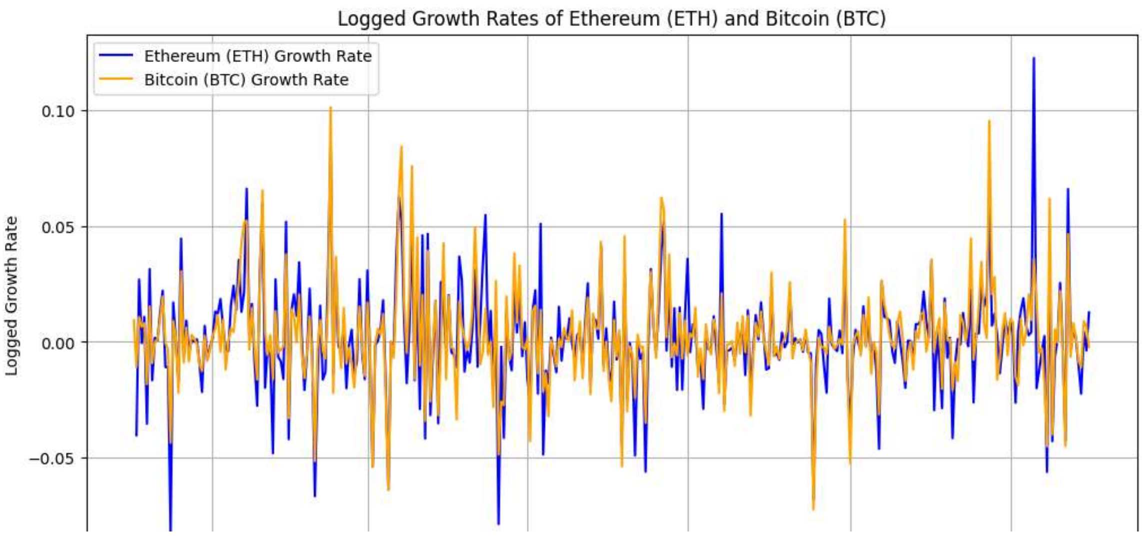
In [132…

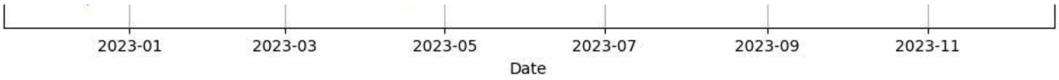
```
#!pip install yfinance
import yfinance as yf
import datetime as dt
import pandas as pd

start = dt.datetime(2022,12,1)
end = dt.datetime(2023,12,2)

df_btc = yf.download('BTC-AUD', start, end)
```

```
[*********************100%%**********************]  1 of 1 completed
```

In [135…

```
import matplotlib.pyplot as plt
import numpy as np

# Assuming df is your Ethereum (ETH) DataFrame and df_btc is your Bitcoin (BT

# Calculate the logged growth rates for ETH and BTC
eth_growth_rate = np.log(df['Close'] / df['Close'].shift(1))
btc_growth_rate = np.log(df_btc['Close'] / df_btc['Close'].shift(1))

# Plot the logged growth rates
plt.figure(figsize=(12, 6))
plt.plot(df.index, eth_growth_rate, label='Ethereum (ETH) Growth Rate', color
plt.plot(df_btc.index, btc_growth_rate, label='Bitcoin (BTC) Growth Rate', co

# Set plot labels and legend
plt.xlabel('Date')
plt.ylabel('Logged Growth Rate')
plt.title('Logged Growth Rates of Ethereum (ETH) and Bitcoin (BTC)')
plt.legend()

# Show the plot
plt.grid()
plt.show()
```





In [138…

```
import numpy as np

# Assuming eth_growth_rate and btc_growth_rate are the logged growth rates
# for Ethereum (ETH) and Bitcoin (BTC), respectively

# Ensure that both series have the same length by removing the first NaN valu
eth_growth_rate = eth_growth_rate.dropna()
btc_growth_rate = btc_growth_rate.dropna()

# Calculate the mean of the Ethereum (ETH) logged growth rates
mean_eth = np.mean(eth_growth_rate)

# Calculate the total sum of squares (TSS)
tss = np.sum((eth_growth_rate - mean_eth) ** 2)

# Calculate the sum of squares of residuals (RSS)
rss = np.sum((eth_growth_rate - btc_growth_rate) ** 2)

# Calculate R-squared (coefficient of determination)
r_squared = 1 - (rss / tss)

print(f'R-squared: {r_squared:.4f}')
```

```
R-squared: 0.7023
```

Looks like the growth rates are overlaid on each other and the R squared is quite high. I think its safe to assume that BTC affects ETH significantly.

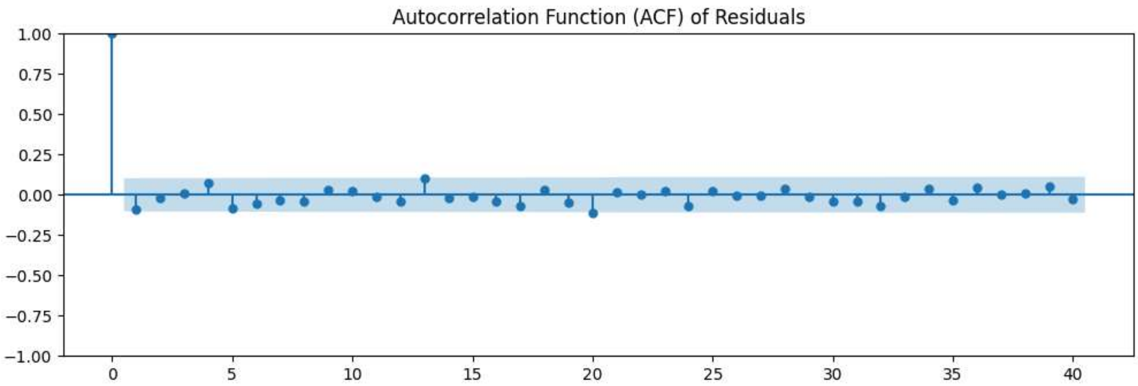
Lets model this by using a simple linear regression of y=X1 * B1 + e and investigating the ACF and PACF of the error term:

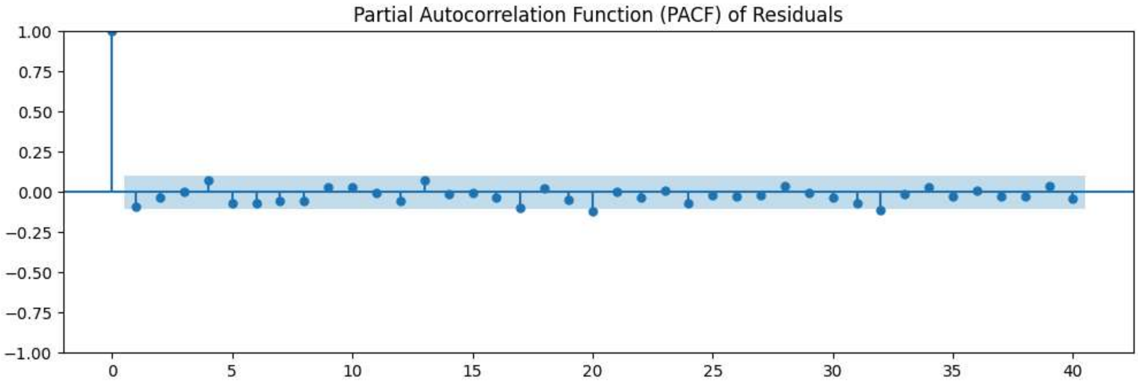
In [142…

```
import statsmodels.api as sm
import pandas as pd

# Assuming eth_growth_rate and btc_growth_rate are the logged growth rates
# for Ethereum (ETH) and Bitcoin (BTC), respectively

# Ensure that both series have the same length by removing the first NaN valu
eth_growth_rate = eth_growth_rate.dropna()
btc_growth_rate = btc_growth_rate.dropna()

# Align the indices of both series
eth_growth_rate.index = pd.to_datetime(eth_growth_rate.index)  # Convert inde
btc_growth_rate.index = pd.to_datetime(btc_growth_rate.index)  # Convert inde
eth_growth_rate, btc_growth_rate = eth_growth_rate.align(btc_growth_rate, joi

# Fit the linear regression model using logged growth rates
X = sm.add_constant(btc_growth_rate)
model = sm.OLS(eth_growth_rate, X)
results = model.fit()

# Print the summary of the regression results
print(results.summary())

# Calculate the residuals of the regression model
residuals = results.resid
```

```
# Plot the ACF and PACF of the residuals
fig, (ax1, ax2) = plt.subplots(2, 1, figsize=(12, 8))

# ACF plot
sm.graphics.tsa.plot_acf(residuals, lags=40, ax=ax1)
ax1.set_title('Autocorrelation Function (ACF) of Residuals')

# PACF plot
sm.graphics.tsa.plot_pacf(residuals, lags=40, ax=ax2)
ax2.set_title('Partial Autocorrelation Function (PACF) of Residuals')

plt.show()
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                  Close   R-squared:                       0.710
Model:                            OLS   Adj. R-squared:                  0.709
Method:                 Least Squares   F-statistic:                     886.7
Date:                Mon, 25 Dec 2023   Prob (F-statistic):           2.30e-99
Time:                        16:40:38   Log-Likelihood:                 1074.0
No. Observations:                 364   AIC:                            -2144.
Df Residuals:                     362   BIC:                            -2136.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const         -0.0007      0.001     -1.067      0.287      -0.002       0.001
Close          0.9138      0.031     29.778      0.000       0.853       0.974
==============================================================================
Omnibus:                      114.878   Durbin-Watson:                   2.169
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              982.146
Skew:                           1.062   Prob(JB):                    5.37e-214
Kurtosis:                      10.762   Cond. No.                         46.1
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctl
y specified.
```





The BTC coefficient is significant. Therefore, for every 1% increase in BTC, there is a 91.38% increase in ETH.

Further, the ACF and PACF of the residuals show there is no autocorelation left in the lags.